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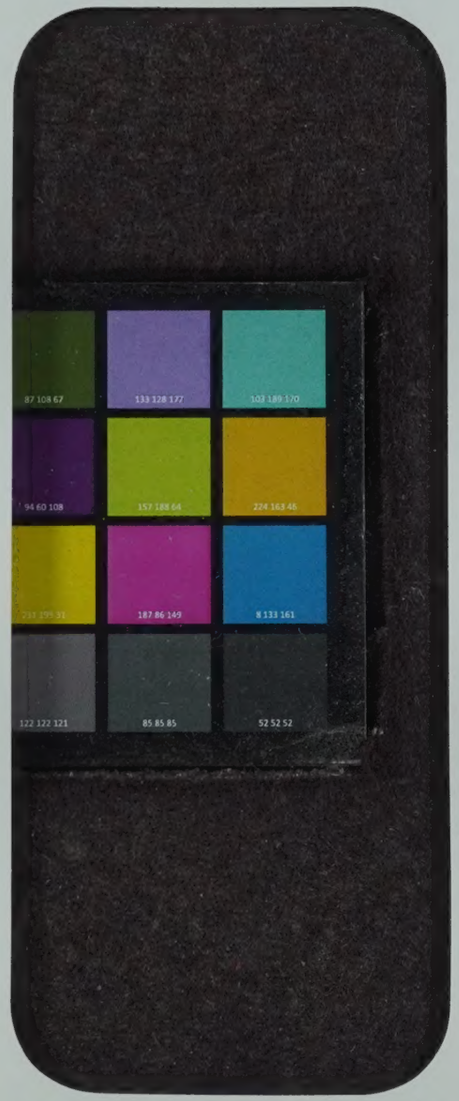
87 108 67
133 128 177
103 189 170
94 60 108
157 188 64
224 163 46
187 86 149
8 133 161
122 122 121
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52 52 52

THE PREPARATION AND TYPING OF MATHEMATICAL MANUSCRIPTS

Second Revised Edition

Copies of this manual may be obtained without charge

by writing to:

Mathematical Manuscripts
Bell Laboratories
600 Mountain Avenue
Murray Hill, New Jersey 07974

PREFACE TO 1973 EDITION

Earlier editions of this manual assumed that the typist would use a standard electric typewriter, such as the IBM Model D, with 100 or more interchangeable keys for mathematical and other special symbols. This edition also provides for the use of IBM Selectric ("golf ball") typewriters having a somewhat smaller number of special characters. In the few places where different instructions are necessary for a Selectric machine, the Selectric instructions are noted immediately following those for a standard machine.

When the manual was originally written, we did not know of any comparably detailed guide which could be used to train typists to set up complicated mathematical material entirely on a typewriter. Since then a very good publication has appeared which fills this gap; it is Guide To Technical Typing, by Rita de Clercq Zubli and Cynthia B. Wong (General Learning Corp., Morristown, New Jersey, 1969; ix + 251 pp.). Zubli and Wong's book includes practically everything that is in the present manual about mathematical typing, plus a great deal more on the use of the typewriter, on chemical notation, on technical illustrations, and on setting up technical reports and manuscripts. It is an excellent reference for anyone who is studying technical typing either alone or in a group.

FOREWORD

To produce attractive and readable typewritten mathematics requires close cooperation between the author of the manuscript and the typist. This manual, developed at Bell Laboratories, contains recommendations to both authors and typists, which it is hoped will provide a basis of understanding for such cooperation.

The first section is directed toward authors; typists need not be concerned with this part. The task of the typist, and also of the ultimate reader, may be greatly simplified if the author bears in mind the limitations of the typewriter, and follows a few simple suggestions in setting up mathematical expressions. The author's duty to explain clearly what he wants is also stressed.

The second part describes for the typist certain conventions that are widely accepted in the typing of mathematics. The present recommendations should be sufficient to guide the typist in most ordinary situations. Specific instructions by the author of a particular manuscript take precedence, of course, over general rules.

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I. PREPARATION OF MANUSCRIPTS

Nothing in this manual is intended to restrict the prerogative of every author to write what he likes and to have it typed according to his instructions. But when an author brings a mathematical manuscript to be typed, he is obligated (1) to know exactly what he wants, (2) to be sure that this is within the capabilities of a typewriter, and (3) to make his wishes clearly understood by the typist.

1. What Do You Want?

Individual preferences as to the form of typewritten mathematics vary considerably. In fact, there is almost no rule which would be universally accepted by everyone who occasionally uses a mathematical expression in a typewritten document. To simplify the training of mathematically inexperienced typists, definite rules are given in the second part of this manual on various points which are actually more or less matters of taste. The present conventions are believed to be acceptable to most professional mathematicians and to mathematical printers.[1]

Typists are under instructions to follow the rules given in this manual except when otherwise instructed by the author of a particular manuscript.

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[1] T. W. Chaundy, P. R. Barrett, and Charles Batey, The Printing of Mathematics, Oxford University Press, London, 1957. This felicitously-written little book is highly recommended to mathematical authors.

It therefore behooves each author to read the manual and to note where his preference differs from what is expressed in it. He will then be able to inform the Transcription Supervisor or Coordinator just how to deviate from the manual in typing his job. Various details about which an author may wish to make his own decision are mentioned in Section 3 below.

2. Consider the Typewriter

In preparing a mathematical manuscript, it is very important to remember the limitations of the typewriter. A typist does not have the resources of a printer with regard to form, size, and spacing of symbols; and even a printer cannot simulate in type everything that can be written with a pen. By keeping in mind the ultimate appearance of the typed or printed symbols, it is often possible to arrange mathematical expressions in manuscript so as to maximize the attractiveness and legibility of the end result.

It is generally desirable to limit the vertical height of expressions which involve several different levels of type. "Slant fractions" (i.e., fractions employing the solidus) can help here. They should always be used when a fraction appears in the text, as a/b, dy/dx, or (x+y)/(x-y). Similarly, if a display formula involves vertical fractions, such as

$$\frac{\frac{x+a}{x+b}+\frac{x+c}{x+d}}{x+y+1},$$

the author should consider using a more easily typed (and printed) form with slant fractions, for example,

$$\frac{(x+a)/(x+b) + (x+c)/(x+d)}{x+y+1} .$$

The use of the abbreviation "exp" for the exponential function is very convenient when such a function appears in the text, or in a display formula with an exponent involving superscripts or subscripts. Also, the radical sign $\sqrt{\ }$ is difficult to space properly when the radicand includes superscripts or subscripts. Usually a fractional exponent makes a neater expression. For example, instead of

$$\frac{e^{x^2+y^2+z^2}}{\sqrt{x^2+y^2+z^2}} ,$$

one should consider using

$$\frac{\exp(x^2+y^2+z^2)}{(x^2+y^2+z^2)^{\frac{1}{2}}} ,$$

or even

$$(x^2+y^2+z^2)^{-\frac{1}{2}}\exp(x^2+y^2+z^2).$$

When a complicated expression is rearranged to reduce its vertical height, one should always be careful to avoid ambiguity by suitable bracketing. Of course, it

is the author's responsibility to set up his mathematics exactly as it is to be typed, since the typist is never supposed to make any changes in the form of a mathematical expression.

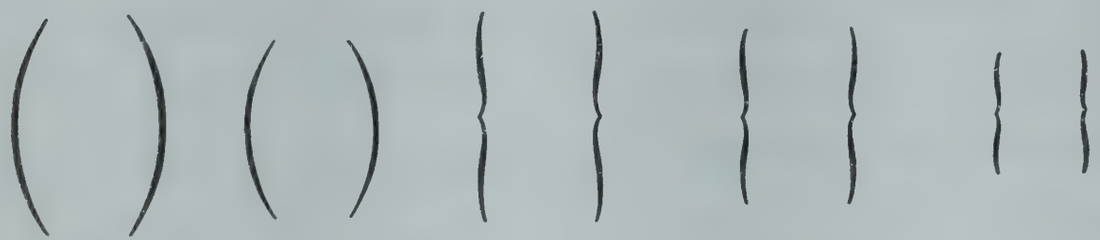
Subscripts and superscripts present occasional pitfalls to the unwary. Small numerals are available for purely numerical subscripts and superscripts, but letter subscripts and superscripts are necessarily the same size as the symbols to which they are attached. It follows that expressions which involve superscripts on superscripts and the like may become rather cumbersome. If such notation is unavoidable, the desired spacing should be made very clear to the typist. Furthermore, text material containing many superscripts or subscripts should not be single spaced. One-and-a-half spacing provides a reasonable compromise between compactness and legibility.

The use of the numeral 1 as a superscript is so much less common than the use of a prime that an author who actually wants a ^{1} superscript should make a special note of the fact. Be careful also to make a clear distinction between a subscript $_{1}$ and a comma.

An author should always write lengthy equations in a manuscript exactly the way he hopes to see them typed, having due regard for the maximum number of typewriter spaces in a line. A line of pica type which exactly fills the space between the usual margins contains 57 spaces.

A line of elite type of the same length contains 67 spaces; and as a last resort, typists are allowed to break each margin by three or four spaces when setting a long equation in elite. As a general rule, however, it is better for an author to use short lines and more of them, while dividing his equations in a sensible way, than to force decisions on a typist who is without mathematical training.

Another matter requiring forethought concerns the bracketing of complicated expressions. Double-size parentheses, brackets, and braces which will enclose three lines are available for the IBM Model D typewriter on special keys. Large square brackets can be produced on a typewriter in any desired size. For very large parentheses or braces, transfers are available in varying sizes; for example:



The application of transfers is so time-consuming, however, that they are used only on final originals. For preliminary drafts and carbon copies, the author should say whether he expects the typist to draw the large enclosing symbols herself by hand, or to leave space for him to draw them.

3. Make It Clear

Legible handwriting is the first requisite for accurate transcribing of any manuscript. This is especially true of technical documents, in which the context may be of little help to the typist confronted by an illegible word. Words which are not likely to be a part of the typist's general vocabulary, or which are used in an unfamiliar context with a special sense, should be clearly printed, at least the first time they occur.

Furthermore, it is the duty of every mathematical author to identify for the typist all Greek letters and all special symbols. The symbols which are available on the IBM Model D in general use at Bell Laboratories are listed by name on pages 55-57 of the manual. *Selectric symbols are shown on pages 58-59.* The easiest way to identify such a symbol for the typist is to insert the correct name near it. Any specially constructed symbol (for example, $\emptyset$ if it is not to be typed as φ) should also be carefully identified on first appearance, and thereafter whenever ambiguity is possible.

Finally, upper and lower case roman letters which have similar shapes, such as S and s, C and c, W and w, or X and x, should be clearly differentiated in mathematical expressions.

When an author brings a mathematical manuscript to be typed, he should note any matters of style about which he feels strongly, since there are a few instances in which tastes differ widely. The most frequent questions have to do with the spacing of symbols in mathematical equations. Spacing is discussed on pages 23-34 of the manual; but it must be realized that in no aspect of mathematical typing do individual preference, judgment, and experience play a more important role.

With a few clearly defined exceptions, printers leave no extra spaces around the operation symbols +, -, ×, ·, ÷, /. However, printers' type is wide enough so that it automatically shows more space around these symbols than a typewriter does. Hence most authors feel that, in typescript, extra spaces are sometimes appropriate and sometimes not.

This manual instructs typists to leave no space around the symbols ×, ·, ÷, /, except when they are used with built-up bar fractions. Multiplication and division are felt to link mathematical quantities more closely, in a sense, than addition and subtraction, in view of the convention that when a sequence of mathematical operations

is written without parentheses, the multiplications and divisions are to be carried out before the additions and subtractions.

In general, space is to be left around + and - signs, because if it is not so left in complicated expressions, the work looks crowded to most authors. For simple expressions there is a difference of opinion about the relative merits of open and closed spacing; for example,

$$x^2 + y^2 - z^2 = 1 \quad \text{or} \quad x^2+y^2-z^2 = 1.$$

This manual calls for open spacing as long as there is plenty of room on the line. Some condensation, expecially of expressions enclosed in parentheses, is permissible in order to avoid breaking an equation awkwardly between two lines. Completely closed spacing of simple equations, as in the above example on the right, is not in any way incorrect; but an author who wants this style on his job is expected to ask for it.

A few authors want a double space left on either side of mathematical symbols such as x or $3c$ and expressions such as $\cos z$ within the text only. The extra spaces are in lieu of italics, which are generally used in printing mathematical symbols; also they prevent confusion of the word "a" and the symbol a. An author can get this spacing by making a special request for it, and by being sure that his manuscript clearly indicates the desired style.

Other items about which an author may wish to indicate a preference include the following:

(a) Special symbols differing from those specified by the manual; for example, $\geqq$ instead of $\geq$ in inequalities.

(b) Zero subscripts; for example, $J_0(x)$ or $J_o(x)$.

IMPORTANT

When a manuscript is finished, spend a short time looking it over for possible obscurities, before the typist has a chance to misread them. This may save hours of proofreading and correcting and may greatly improve the quality of the final job.

II. GUIDE FOR MATHEMATICAL TYPING

The typing of mathematics requires a much higher degree of skill than the typing of ordinary text. A good mathematical typist must be an artist at translating handwritten symbols into typescript which quickly and clearly conveys the author's meaning to the reader. To a certain extent mathematical typing, like any art, can be taught; but complete mastery comes only with experience.

This manual is a style guide which describes certain conventions that are widely accepted in the typing of mathematics. It will cover most ordinary situations and will be acceptable to the majority of mathematical authors. Typists, stenographers, and secretaries are expected to be familiar with the manual, and to follow it unless otherwise instructed by the author of a particular manuscript. It must be realized, however, that in view of the complexity of modern mathematical notation and the individual preferences of different authors, no rigid set of rules can apply to every possible contingency. An author who wishes to use symbolism not given in the manual, or to deviate from the style recommended here, is expected to include special instructions with his manuscript. The author's instructions always take precedence over the manual. If the manuscript is ambiguous, call the author for clarification. In any case, it is a good idea to look over the whole job before typing, in order to see what is coming up.

Mathematical typing inevitably takes a great deal more time per page than straight text. An attempt to hurry the setting up of a complicated formula is very likely to lead to unsatisfactory results. A paramount rule in mathematical typing is therefore: Always take time to do a good job. A reasonable amount of speed will come with experience; but speed is never to be sought at the expense of quality.

1. Special Symbols

The mathematical typist must always remember that to a mathematician a symbol is not just a letter or a collection of characters; to him it stands for a whole concept, which by itself might require several sentences to define. Inadvertent substitution of one symbol for another (even upper case for lower case or vice versa) may thus render a whole formula meaningless or, still worse, incorrect. Since there are many more concepts than there are simple roman letters, mathematical authors are forced to use many special symbols. Some of the more common symbols are discussed in this section. Each author should identify his special symbols at the beginning of his manuscript; but if the typist still does not recognize a particular symbol, she should check with the author before typing it.

In order to communicate easily with authors, a competent mathematical typist needs to be familiar with mathematical terminology. For this reason, a list of

mathematical symbols and special characters, with their names, is provided on pages 55-57. Examples of other mathematical terminology will be found throughout the manual. The beginning typist is not expected to memorize all of these terms or to understand their mathematical meaning. Some of them occur only in very specialized work, and she will soon become acquainted with the ones which appear most often on her own jobs.

(a) Greek Letters

Care should be taken not to confuse Greek and non-Greek characters. One must distinguish carefully among the following letters and symbols:

a, α (alpha), ∞ (infinity), and $\propto$ (proportional)

d and ∂ (alpha)

K, k and κ (kappa)

u and μ (mu)

v, ν (nu), γ (gamma), 2, and υ (upsilon)

w and ω (omega)

X, x, χ (chi), and $\times$ (times)

x and ψ (psi)

When a Greek letter is called for, do not improvise; always use the Greek key. For example, $\emptyset$ (0 plus solidus) is not to be used for φ unless the author specifically requests $\emptyset$. (The symbol $\emptyset$ is used for typing IBM computer codes and in a few other mathematical contexts.)

(b) Embellished Characters

Mathematicians may embellish symbols by placing above or below them various combinations of dots, bars, tildes, circumflexes, and arrows.

$$\dot{x},\ \ddot{x},\ \bar{x},\ \tilde{x},\ \hat{x},\ \vec{x},\ \underline{x},\ \underset{\sim}{x}$$

The embellishment should be placed as close as possible to the original character without actually touching it. Some embellishments have to be hand rolled to the correct position, depending on the height of the original character.

Good	Poor
$\vec{E}$	$\vec{E}\quad\overrightarrow{E}$

Asterisks are frequently attached to mathematical symbols, as z*, I*. When so used, they do not refer to footnotes! When an asterisk is used with a subscript, raise the asterisk a half space: z_1^*.

(c) Parentheses, Brackets, Braces

Mathematical authors generally use three kinds of enclosing symbols: parentheses ( ), brackets [ ], and braces { }. Angular brackets $\langle\ \rangle$ occur less frequently, and almost always have a special meaning.

(i) Parentheses, brackets, and braces usually occur in pairs.

(ii) One set of enclosing symbols may enclose another set of the same size, provided that the two sets of symbols are of different kinds. If one set encloses another set <u>of the same kind</u> in an expression which is complicated enough to be displayed on a separate line, the outer set should be larger than the inner set.

$$[a + (b^2-c^2)^{\frac{1}{2}}] \qquad \left(a + (b^2-c^2)^{\frac{1}{2}}\right)$$

However, the nesting of small parentheses, brackets, or braces in expressions in running text, such as $\sin(\omega t + \varphi(t))$, is acceptable.

(iii) Simple expressions containing subscripts or superscripts do not require large enclosing symbols.

$$(n^2-a_1) \qquad [n^2-a_1] \qquad \{n^2-a_1\}$$

(iv) Use double-size parentheses, brackets, or braces for horizontal bar fractions whose vertical height does not exceed three lines, and for expressions with both superscripts and subscripts on the same term unless the author has requested

small enclosing symbols. Double-size parentheses or braces may in an emergency be used around an expression having a height of three and a half lines; but if brackets are called for around an expression with more than three lines, they should be built up as in (v) below.

$$\left(\frac{1}{\pi}\right) \qquad \left(\frac{n^2-1}{n^2+1}\right) \qquad \left\{\frac{n^2-1}{n^2+1}\right\} \qquad \left[a_2^2 + b_2^2\right] \qquad \left(\frac{a^2}{b^3 c_1}\right)$$

On Selectrics, transfer parentheses, brackets, and braces should be used whenever an expression contains both a superscript and a subscript and when you are enclosing a complicated expression.

$$\left[a_2^2 + b_2^2\right] \qquad \left(\frac{a^2}{b^3 c_1}\right)$$

(v) When it is necessary to bracket a complicated expression, square brackets should be constructed which extend from just above the top superscripts to just below the bottom subscripts (if any), using the underscore and the offset vertical bar on the 7 key. *Use transfers on Selectrics.* This is the only occasion on which the offset vertical bar is used.

Remember to space once before typing the final bar. When numerator and denominator are of unequal height, use the main line as the center of the enclosing marks.

$$\left[\frac{a_1^2}{b^3c_1}\right] \qquad \left[\frac{a_1^2}{bc}\right] \qquad \left[\frac{a}{b^3c_1}\right]$$

(vi) If larger parentheses or braces than in (iv) seem to be called for, use transfers on final originals.

$$\left(\frac{\frac{a_2}{x^2}+\frac{a_1}{x}}{\frac{b_2}{y^2}+\frac{b_1}{y}}\right) \qquad \left\{\frac{\frac{a_2}{x^2}+\frac{a_1}{x}}{\frac{b_2}{y^2}+\frac{b_1}{y}}\right\}$$

(d) Vertical Bars

Pairs of vertical bars are another type of enclosing symbol. Do not confuse them with I's, l's, or /'s. Use the centered vertical bar in cases like the following.

Absolute value

$$|(x^2-y^2) + 2ixy|$$

Determinant

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

The spacing of more complicated determinants is discussed on pages 30-31.

Occasionally pairs of closely spaced double bars (the "parallel" key or transfers) are used as an enclosing symbol.

Norm

$$\|x-y\|$$

A single vertical bar is sometimes used in contexts such as the following (the typist does not need to worry about the names).

Conditional probability

$$P(E_1|E_2E_3)$$

Set definition

$$\{x|f(x) = 1\}$$

Substitution

$$\tan^{-1}\theta\Big|_0^{\infty} \qquad (x^2+3x+2)\Big|_{x=a}$$

The one-sided bracket may also be used as a sign of substitution.

$$\tan^{-1}\theta \Big]_{0}^{\infty}$$

A substitution bar or bracket should always be long enough so that the symbols on the right are clearly above and below the main line.

(e) Integrals, Summations, Products, Radicals

Integral signs ($\int$) should generally be larger, but not much larger, than what follows. All of the integral signs in any one equation should be of the same size. Use the multilith pencil to draw the vertical line.

Good	Poor
$\int \frac{\sin ax}{\cos bx}\, dx$	$\int \frac{\sin ax}{\cos bx}\, dx$
$\int f(x)dx$	$\int f(x)dx$

The key $\int$ is occasionally used with simple expressions in running text, but it is not to be used in an equation which is displayed on a separate line.

Limits of integrals are symbols written at the upper and lower right of the integral sign and are typed to the right of the integral sign. Examples:

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx \qquad\qquad \int_{0}^{\tan y} \frac{\sin x}{x}\,dx$$

The initial symbols of the upper and lower limits should be aligned vertically (+ and - signs are not counted as initial symbols here). The integral sign should be large enough so that the expression following it can stand clear of the limits both vertically and horizontally. Spacing of integrals is further discussed on pages 31-32.

To make contour integral signs ($\oint$, $\oint\!\!\!\!\!\!\subset$) hold the typewriter carriage carefully to center the O or the C, and draw the vertical connecting line with the multilith pencil.

$$\oint \frac{w^2 dw}{w^4+1} \qquad\qquad \oint\!\!\!\!\!\!\subset \frac{f(z)}{g(z)}\,dz$$

Variants of this sign occasionally turn up, so ask the author what he wants unless the manuscript is perfectly clear.

A small summation (Σ) or product (Π) sign may be used if the expression which follows it is simple (e.g., Σa_i), or if it occurs in running text. However,

if the sign is followed by a complicated expression, or if limits are written above and below the sign, use the typewritten $\sum$ or $\prod$ of double height. *On Selectrics use transfers.*

$$\sum \frac{1}{k} \qquad \sum_{n=0}^{\infty} \frac{n^2}{n^4+1} \qquad \prod_{n=1}^{\infty} \left[1 - \frac{x^2}{n^2\pi^2}\right]$$

Do not mix signs of different sizes in the same equation.

The radical or "square root" sign $\sqrt{\ }$ should extend just below the bottom of what follows it, including subscripts, and the bar should be just above the symbols that it covers without touching them. If a figure or symbol is written over the radical sign it should be raised slightly above the bar. Use the multilith pencil to connect the radical sign and the bar smoothly if the expression following the radical is more than one line high; for example, if it has superscripts or subscripts. *Transfers are available on Selectrics for radicals covering more than one line.*

Good	Poor
$\sqrt{x}$	
$\sqrt[n]{x+a}$	
$\sqrt{x_1^2 + x_2^2 + x_3^2}$	$\sqrt{x_1^2 + x_2^2 + x_3^2}$
	$\sqrt{x_1^2 + x_2^2 + x_3^2}$
$\sqrt[3]{\frac{x+1}{x-1}}$	$\sqrt[3]{\frac{x+1}{x-1}}$

If a radical sign occurs in the denominator of a horizontal fraction, leave one-half space between the fraction bar and the radical bar.

$$\frac{x+y}{\sqrt{x^2+y^2}}$$

Leave an extra space before a radical sign $\sqrt{\ }$ which appears in the text.

(f) Miscellaneous

Some authors write the letters Z, z with a bar to distinguish them from the numeral 2. Do not type the bar unless specifically requested.

Written	Typed
$\bar{Z}, \bar{z}$	Z, z

The script letter ℓ is used in mathematical expressions where an ordinary typewritten letter would be confused with the number 1. The script ℓ should not be used as the first letter of a word or abbreviation.

Primes (',") should not be confused with 1's. (The use of 1 as a single superscript is so uncommon that an author who intentionally uses a raised ^{1} is expected to call attention to the fact.) Use the prime key for primes. *The prime key is not available on the Selectric; therefore use of the apostrophe or quotation mark is necessary.*

Model D: x', x'' *Selectric:* x', x"

When a subscript is used with a prime, the prime should be typed one-half space above the main line.

Model D: x_2' *Selectric:* x_2'

Care should be taken to distinguish 0 (zero) from the small letter o in limits of integrals and summations, and in subscripts and superscripts.

$$\int_0^1 J_0(x)dx \qquad \sum_{n=0}^{\infty} \frac{1}{n^3+1}$$

The symbol 0 is almost invariably correct in subscripts and superscripts, and must always be used if other numerals appear in the same context.

Right	Wrong
$x^{(0)}$, $x^{(1)}$, $x^{(2)}$	$x^{(o)}$, $x^{(1)}$, $x^{(2)}$
TE_{03}, λ_{03}	TE_{o3}, λ_{o3}

If the symbol appears as a subscript or superscript either alone or associated only with letter symbols (for example, δ_{0n}), some authors prefer the small o because of its compactness (δ_{on}); but if possible, the author should be consulted before the job is typed.

The times sign × is not to be improvised by x, since x is used very frequently as an algebraic symbol.

The centered dot usually means multiplication. If a centered dot key is not available, then the dot is to be centered by hand-rolling the carriage. Occasionally such a dot stands alone between two parentheses, in lieu of some other mathematical symbol.

Good	Poor
$A \cdot B$ $f(\cdot)$	$A.B$ $f(^{\bullet})$

2. Spacing of Mathematical Symbols

Typewritten mathematical work should not be unduly crowded. Each symbol, and the relations among the symbols, should be distinctly visible. On the other hand, the typing should not be so loose as to cause doubts as to whether or not something has been omitted, or to necessitate an effort on the part of the reader to grasp what is being said.

Mathematical typing should be double spaced, with indented paragraphs, unless the author has given specific instructions to the contrary. If he wants a compact job but the text contains subscripts and superscripts, suggest one-and-a-half spacing.

(a) No Sign - No Space

Letter or number symbols written with no sign between them should not be spaced.

$$3i\omega\pi n^2 ab \qquad 5Ie^{i\omega t}$$

No space should be left between a mathematical expression in parentheses, brackets, or braces and letter or number symbols immediately preceding it (except, as on page 38, when the preceding symbols are a built-up bar fraction).

$$3a^2(a+b) \qquad f(x) \qquad G[x,y]$$

Note, however, that if words or assertion symbols (see below) are enclosed in parentheses, the parentheses are to be preceded and followed by a space. Examples: "Let R (the radius of the sphere) be a slowly varying function ..." "Choose x. ($x^2 < 1$) to satisfy the equation ...".

An apparent exception to the rule that letter symbols are not spaced occurs if two or more letters together represent a mathematical abbreviation, such as "sin" or "dx". Mathematical abbreviations are discussed in Section 2(d) below.

(b) Equality and Inequality Signs (Assertion Symbols)

In general, a space should be left on each side of equality and inequality signs, such as

$$=,\ \equiv,\ \neq,\ >,\ \geq,\ <,\ \leq,\ >>,\ <<,\ \approx,\ \simeq,\ \doteq,\ \sim.$$

Exception

Do not space out equality signs in limits of summations and products, or with substitution signs (pages 17-18).

$$\sum_{i=0}^{n} \quad \prod_{n=0}^{\infty} \quad (x^2+3x+2)\Big|_{x=a}$$

When used in a mathematical or a chemical expression, the arrow sign ($\rightarrow$ or $\leftarrow$) requires a space on both sides.

Chemical	Mathematical
$HCl + NaOH \rightarrow H_2O \rightarrow NaCl$	$T \rightarrow T_c$

The arrow sign is not spaced out when it occurs below the abbreviation "lim".

$$\lim_{x\rightarrow 0} \frac{\sin x}{x} = 1$$

The various implication signs must be improvised from equality and inequality signs (*transfers on Selectrics*), and require a space on both sides.

$$A \Rightarrow B \qquad P \Leftarrow Q \qquad X \Longleftrightarrow Y$$

The symbol ϵ, which is read "is an element of" or "belongs to", is spaced like an equality sign. *On Selectrics, an epsilon ε may be used with the author's permission.*

$$\alpha \in A$$

(c) Mathematical Operations (+, -, ×, ÷)

Leave no space around the multiplication and division symbols ×, ·, ÷, /, except when they occur next to a built-up horizontal bar fraction (page 38).

Leave no space around the symbols + and - if they connect simple expressions grouped inside a pair of parentheses, brackets, or braces. Space may also be omitted if the + or - sign connects simple expressions which are implicitly grouped, for example, in the numerator or denominator of a fraction, or in subscripts and superscripts.

$$(x+iy) \qquad [a^2-4] \qquad \frac{n-4}{n^2+3n+2} \qquad r^{n+2}$$

Do leave a space on each side of a + sign or - sign which connects complicated expressions, or any expressions which are not enclosed in parentheses or otherwise particularly grouped. An example of a complicated expression would be one involving both superscripts

and subscripts. If this rule requires spacing around signs inside parentheses, then all signs within the same parentheses should be spaced in the same way.

$$x_1^2 + y_1^2 + z_1^2 \qquad \left(a + b + c^3 + d_1^2\right)$$

Leave spaces around a + or - sign before and after parentheses, brackets, or braces (except in superscripts and subscripts, as on page 36).

$$a + (a^2+b^2) - c^2 \qquad a + b(c+d) \qquad [a + (a^2+b^2) - c^2]q$$

NOTE: Open vs. Closed Spacing

Mathematical writers do not always agree about the spacing of equations. A frequent point of disagreement is whether to leave spaces around + and - signs between simple expressions which are grouped, not by enclosing symbols, but merely by being on the same side of an equation. In equations such as the following some authors prefer one style and some the other.

Open	Closed
$x + y + z = a + b$	$x+y+z = a+b$
$r^2 - t^2 = 1$	$r^2-t^2 = 1$

Open spacing should be used between simple expressions on the same side of an equation, provided that there is plenty of room and the author has not

given specific instructions for closed spacing. However, if a long equation containing simple expressions can all be typed on one line by omitting a few spaces, it is often better to do this than to spread out the equation and have to break it awkwardly between two lines.

In this manual the equations are all open spaced, even though closed spacing would not be incorrect in examples like the above. If an author requests closed spacing, then the closed style should be followed as consistently as possible throughout his job.

NOTE: Positive and Negative Numbers

Do not leave a space after a + or - sign which merely indicates that the symbol following it is a positive or negative number. Examples: "The allowable frequency variation is ±10 kHz." "The charge on the ion is -2e."

The sign ~ is sometimes used before a number or letter symbol to mean "approximately". In such cases it should not be spaced out, and it should be hand rolled to the center of the line. Example: "The mean thermal energy is ~0.03 eV."

(d) Mathematical Abbreviations

Mathematical abbreviations should ordinarily be preceded and followed by a space. Frequently used abbreviations include:

sin	arcsin	sinh	exp	sn	sgn
cos	arccos	cosh	log	cn	sup
tan	arctan	tanh	ln	dn	inf
cot	$\sin^{-1}$	coth	lim	Pr	
sec	$\cos^{-1}$	sech	Re	cov	
csc	$\tan^{-1}$	csch	Im	erfc	

Examples are:

$$\cos az \qquad \log x^n \qquad \lim f(x)$$

Exception

Before or after a superscript, subscript, or enclosing symbol the space is unnecessary.

$$e^{ax}\sin bx \qquad (n+1)\tan A \qquad \tanh(x+iy) \qquad \log_e x$$

(e) Differentials and Partial Differentials

A differential, such as dx, dS, δy, or δW, is really a mathematical abbreviation and should ordinarily be preceded and followed by a space, except when it follows a superscript, a subscript, or an expression within parentheses.

$$fg\ dx\ dy\ dz \qquad t^n dt \qquad p(x)\delta x$$

An author who wants all his differentials unspaced should make a specific request to this effect with his manuscript.

A partial differential (denoted by the sign ∂) occurring in the denominator of an expression should not be spaced out.

$$\frac{\partial^2 f}{\partial x \partial y}$$

(f) Determinants and Matrices

These are two-dimensional, square or rectangular arrays of mathematical symbols enclosed by vertical bars (determinants) or by large brackets or parentheses (matrices).

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\gamma & 1 - \tfrac{1}{2}\gamma^2 & -\tfrac{1}{2}\gamma^2 \\ 0 & \alpha - \gamma & \tfrac{1}{2}(\alpha - \gamma^{-1})^2 & 1 + \gamma^2 - \alpha \\ 0 & \tfrac{1}{2}(\alpha - \gamma^2)^{-1} & \dfrac{1+\gamma^2-2}{\alpha-\gamma} & \dfrac{1+\frac{1}{2}\gamma^2}{\gamma+\alpha^2} \end{bmatrix}$$

Spacing may be a problem if the elements of the array are of markedly different sizes. In such a case the typist should imagine the space between the enclosing symbols to be divided by horizontal and vertical lines into rectangular boxes, with each element centered in its own box thus:

$$\left[\begin{array}{c|c|c|c} 1 & 0 & 0 & 0 \\ \hline 0 & -\gamma & 1-\frac{1}{2}\gamma^2 & -\frac{1}{2}\gamma^2 \\ \hline 0 & \alpha-\gamma & \frac{1}{2}(\alpha-\gamma^{-1})^2 & 1+\gamma^2-\alpha \\ \hline 0 & \frac{1}{2}(\alpha-\gamma^2)^{-1} & \frac{1+\gamma^2-2}{\alpha-\gamma} & \frac{1+\frac{1}{2}\gamma^2}{\gamma+\alpha^2} \end{array}\right]$$

Although the partitioning lines are not actually drawn, the spacing must make it obvious that such lines could be drawn.

(g) Integral, Summation, and Product Signs

(i) If an integral sign is preceded by a symbol, leave one space before typing the integral sign.

$$a\int_0^t n(\tau)d\tau$$

(ii) If an integral sign is preceded by a + or - or = sign or by a built-up bar fraction, leave two spaces before typing the integral sign.

$$r = \int_1^t n(t)dt \qquad b = -\int_0^\pi \cos x\, dx \qquad p(t) = \frac{1}{t}\int_1^t \frac{d\tau}{\tau}$$

(iii) Ordinarily no space should be left after typing the longer limit before starting the main line; but if a built-up fraction follows the limits, one space should be left.

$$\int_0^\pi e^{-x}dx \qquad \int_x^\infty t/(bt+c)^3 dt \qquad \int_1^{\exp x} \frac{1}{t}\, dt$$

$$\int e^{-x}dx \qquad \int \frac{1}{t}\, dt$$

(iv) If a summation or product sign has no limits or only single-symbol limits, one space should be left between the sign and preceding or following symbols (including + and - signs).

$$3\sum e^{-n} \qquad -\prod_j (1+a_j)$$

(v) If a large $\sum$ or $\prod$ with no limits or only single symbol limits is preceded or followed by a built-up bar fraction or an equals sign, leave two spaces between the $\sum$ or $\prod$ and the fraction or equals sign.

$$\frac{\pi^2}{6} = \sum \frac{1}{n^2}$$

(vi) In general, if a summation or product is preceded by a character or sign, one space is left before starting the longer limit, and one space is left after the longer limit before typing the next character on the main line. Always center the $\sum$ or $\prod$ with respect to the longer limit.

$$S = - \sum_{i=1}^{[n/2]} \frac{1}{i^2} \qquad P(x) = \prod_{n=1}^{\infty} \left(1 + \frac{x^2}{n^2\pi^2}\right)$$

(h) Miscellaneous

Commas are sometimes used to separate symbols enclosed in parentheses. Usually no space is needed after the comma, but if there are complicated combinations of symbols a space should be left. Examples:

$f(x,y,z)$ coordinates (r,θ) $G(x^{2/3}, y^{7/8}, z^{1/6})$

Space should be left after commas which separate numbers or symbols appearing in ordinary text, when these are not enclosed in parentheses or part of an equation. Example: "Let p, q, r be relatively prime ...". But if there is an = sign the spaces are omitted: "... where i = 1,2,3,...".

A semicolon in a grouped expression should be followed by a space.

Leave two spaces between a mathematical formula and the name or abbreviation of its physical units.

$$v = (2gh)^{\frac{1}{2}} \text{ cm/sec} \qquad \Delta f = \frac{1}{n\Delta t} \text{ Hz}$$

Large parentheses occurring back to back $)($ should be spaced so that they do not touch.

Occasionally an author requests double spaces around mathematical symbols such as $2a$ and expressions like $\tanh \theta$ within the text only. In such cases the typist should, as usual, follow the instructions submitted with the particular manuscript.

3. Superscripts, Subscripts, and Fractions

The principal problem created by superscripts, subscripts, and complicated fractions is one of vertical spacing.

(a) Superscripts and Subscripts

Superscripts and subscripts are indicated in manuscript by small raised or lowered symbols (less

frequently by carets). Inasmuch as some authors merely write subscripts smaller than normal but do not drop them below the line, the typist should question all doubtful cases!

Written	Typed
x^2, x^2	x^2
v_{mn}, v_{mn}, v_{mn}	v_{mn}

If the symbol carries both a superscript and a subscript, it is preferable in most cases to type the superscript directly above the subscript, as in the following example on the left. If, however, the author indicates that the superscript should follow the subscript, it would appear as in the example at the right.

$$x_1^2 \qquad x_1{}^2$$

Authors whose work involves only numerical superscripts and subscripts may ask for small raised and lowered digits.

$$x^2 + y^2 + z^2 = 1 \qquad x_1 + x_2 + x_3 = 0$$

Use the small figures only on request; in no case mix small figures with letter symbols of normal size.

Good	Poor
x^{2a}	x^{2^a}
Z_{m+3}	Z_{m+3}

In a superscript or subscript, do not space between a + or - sign and enclosing symbols.

$$e^{-(n^2+1)} \qquad A_{m-(n+2)}$$

(b) Superscripts Outside Parentheses

If a superscript follows a small closing parenthesis, bracket, or brace, raise the superscript only a half space above the main line.

$$(x^2+y^2)^3$$

If a superscript follows a large bracket or other enclosing symbol, the superscript should be raised a half space above the upper right-hand corner of the bracket.

$$\left[x_1^2 + y_1^2\right]^3$$

When a superscript follows a large built-up bracket and the next symbol is unspaced, hold the typewriter carriage back a half space by hand while typing the superscript. Otherwise the spacing will be awkward,

since the vertical bar used in constructing the bracket strikes slightly to the left. The same rule holds for subscripts.

$$\left[\frac{x_1^2 + y_1^2}{x_1^2 - y_1^2}\right]^3 (z+c)$$

(c) Subscripts on Superscripts et al.

An ordinary superscript is raised a half space above the line, but a superscript which itself bears a subscript should be raised a full space, and the subscript a half space.

$$x^{3n} \qquad \frac{x^{n_1} + x^{n_2}}{x^{n_1} - x^{n_2}}$$

Subscripts which carry superscripts are treated similarly.

(d) Spacing of Fractions

Long bar fractions involving superscripts and subscripts should be carefully read and proper allowance made in typing the bar, so that it will not cut through any symbols. The bar should be exactly as long as the longer of numerator and denominator, including the subscripts and superscripts of either.

$$\frac{x_1}{(x_1+y_1)^2}$$

If the numerator and denominator of a bar fraction are of unequal lengths, center the shorter part with respect to the longer. For short fractions it may be necessary to hold the typewriter carriage by hand at half spaces in order to get perfect centering.

$$\frac{1}{36} \qquad \frac{\partial^2 f}{\partial x \partial y} \qquad \frac{c+d+e}{ab}$$

Half spacing is unnecessary if the whole fraction is more than five spaces long.

Always space before and after a built-up horizontal bar fraction. A key fraction is treated like any other number.

$$ab^2 \, \frac{Ka}{2b} \qquad ab + \tfrac{1}{2}c$$

Note, however, that only two spaces should be left between bar fractions connected by a multiplication dot, and the dot should be centered by holding the carriage.

$$\frac{n-4}{n^2+3n} \cdot \frac{x^2}{y^3}$$

Do not convert a vertical fraction to a slant fraction on your own initiative, since this may change the meaning of the whole expression and make it incorrect or, at best, ambiguous. In particular, do not use a

slant fraction before an enclosing symbol without consulting the author. Use a key fraction, if available, before a small enclosing symbol, and a built-up bar fraction before a large one.

Good	Poor
$\frac{2}{9}$ x	2/9x
$\frac{1}{2}(a+b)$	1/2(a+b)
$\frac{1}{2}\left[x_1^2 + x_2^2\right]$	1/2 $\left[x_1^2 + x_2^2\right]$

(e) Fractional Superscripts and Subscripts

A fractional superscript or subscript which is purely numerical should be typed as a key fraction if it is available and if the author has not given other instructions; otherwise, the solidus (/) should be used.

$x^{\frac{1}{2}}$ $x^{3/7}$ $(a+b)^{\frac{1}{3}}$

But do not mix slant fractions and key fractions in the same equation; that is, type $x^{1/2}y^{1/6}$ rather than $x^{\frac{1}{2}}y^{1/6}$. If many slant fractions have to be used as subscripts or superscripts in a particular job, ask the author whether he wants slant fractions for consistency everywhere, even in cases where key fractions are available.

Exception

Always use slant fractions as superscripts or subscripts with large enclosing symbols; for example:

$$\left(a_2^2 + b_2^2\right)^{1/2} \qquad \left[1 - \left(\frac{a^2}{b^3 c_1}\right)^{1/2}\right]^2$$

If a fractional superscript involves letter symbols, it should be typed as nearly as possible the way it appears in the manuscript.

Written	Typed
$e^{\frac{2\pi i}{3}}$	$e^{\frac{2\pi i}{3}}$

It is true that in this example a neater form would be $e^{2\pi i/3}$, but we repeat the warning that a typist must not change a vertical fraction to slant form without consulting the author.

4. Equations - General

(a) Equality and Inequality Signs

Equality is denoted by the "equals" sign =, and occasionally by the "identically equals" sign ≡. When the ≡ sign is called for, use the special key.

Inequalities are treated in just the same way as equalities. The sign may be ≠, >, ≥, <, ≤, >>, <<, or occasionally a more esoteric sign (e.g., $\lesssim$) constructed

by a particular author. Note that the sign for "is not equal to" is $\neq$, not $\not=$. There are several handwritten forms of the sign $\leq$ (meaning "less than or equal to") and the sign $\geq$ (meaning "greater than or equal to"), but only one standard typewritten form for each.

Written	Typed
$\leq$ $\leqslant$ $\leqq$ $\lesseqgtr$	$\leq$
$\geq$ $\geqslant$ $\geqq$ $\gtreqless$	$\geq$

An author who wants the typewritten form $\leqq$ or $\geqq$ should make a special request for it.

There are several "approximately equals" signs; for example, $\approx$, $\simeq$, $\doteq$, $\sim$, and possibly others. Since authors sometimes make subtle distinctions among these signs, they cannot all be replaced by a single form. If the signs must be improvised, they should be made reasonably compact; for example $\underset{\sim}{\sim}$ is too widely spaced to be a good representation of $\approx$.

Finally, the sign $\in$ ("belongs to", or "is an element of") is treated like an equality sign, as on page 26. Do not replace this sign by the Greek letter ε without the author's permission.

(b) Displaying and Centering

An equation, such as

$$\cos 2x = \cos^2 x - \sin^2 x,$$

is generally written on a line by itself. Occasionally a short equation, such as a = b, may be imbedded in the text. If an equation is displayed on a line by itself, it should be preceded and followed by three vertical spaces.

A single equation standing on a line by itself should be centered. If the equation is so long that the centered equation would start flush with the left-hand margin (or break the margin), then the equation should be set in elite type (see page 43) or it should be divided, as described in Section 6.

When two or more equations of different lengths are not separated by any text, align all the = signs vertically, and center the equations as a group.

$$x = a,$$

$$y + z = b + c,$$

$$x - y + z = d.$$

If the equations in this example were separated by any text (even by "and"), each line would be centered separately.

If it is not feasible to align the = signs of a series of equations of greatly differing lengths, the initial symbols of the left-hand sides of the equations should be aligned vertically, and the longest equation should be centered. Example:

$$p' = - \frac{a^{3/2}(1-e^2)^{3/2}}{k^{1/2}(1 + e \cos f)^2} \frac{\partial V}{\partial f} ,$$

$$\beta'' + \beta + 2\iota \cos \theta + 2\Omega \sin i \sin \theta = - \frac{a(1-e^2)}{k(1 + e \cos f)^2} \frac{\partial V}{\partial \beta} .$$

An author to whom the above arrangements are unsatisfactory is expected to give specific instructions with his manuscript. The typist should be aware, however, that a few authors prefer each equation to be individually centered, and she should ask for instructions if the manuscript seems to be following the latter style.

(c) Use of Elite Type

An equation set in pica type must be indented at least one space from each margin; that is, its length must not exceed 55 spaces. Longer equations, up to 65 spaces, can be done in elite type. Of course all of the equations in a group not separated by any text should be done in the same size of type. The following is an example of an equation which may be set on one line in elite, but not in pica.

$$V = - \frac{k}{r} \left[1 + \frac{J}{3} \frac{R^2}{r^2} (1 - 3 \sin^2 \varphi) + \frac{H}{5} \frac{R^3}{r^3} (3 \sin \varphi - 5 \sin^3 \varphi) + \ldots \right]$$

An equation whose length exceeds 65 spaces should ordinarily be divided. In an emergency, however (for example, in the case of a very long bar fraction), an equation in elite type is permitted to break both margins by three or four spaces, since the equation is felt to be adequately set off from the rest of the text by the smaller type.

(d) Vertical Spacing

In setting up complicated equations, vertical spacing is important. If there is a long fraction bar, it may usually be regarded as the center line of the equation.

$$\int_b^1 \frac{a\,dx}{x(x^2-a^2)^{\frac{1}{2}}} = \sin^{-1}\frac{a}{b} - \sin^{-1}a$$

However, in measuring the space to be left between the equation and text and/or another equation, the highest and lowest points in the equation are the guide lines.

Equations requiring two or more lines should have three vertical spaces between the lowest point of the first line and the top of the second and/or consecutive lines.

In the case of a determinant or matrix with elements of various sizes, the elements should be arranged in imaginary rectangular boxes (as on page 31), the enclosing bars or brackets should just cover the

array vertically, and the center of the bars or brackets should be taken as the center of the equation.

$$\underset{\sim}{M} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\gamma & 1 - \frac{1}{2}\gamma^2 & -\frac{1}{2}\gamma^2 \\ 0 & \alpha - \gamma & \frac{1}{2}(\alpha-\gamma^{-1})^2 & 1 + \gamma^2 - \alpha \\ 0 & \frac{1}{2}(\alpha-\gamma^2)^{-1} & \frac{1+\gamma^2-2}{\alpha-\gamma} & \frac{1+\frac{1}{2}\gamma^2}{\gamma+\alpha^2} \end{bmatrix}$$

5. Punctuation of Equations

Mathematical authors punctuate equations for perfectly definite grammatical reasons. Therefore, the typist should follow the manuscript just as carefully in punctuating equations as in punctuating text.

(a) Punctuation Following an Equation

The author who writes "a = b" always reads this in his mind as "a equals b", thinking of the = sign as a verb, and he will accordingly use whatever punctuation is dictated by the grammatical structure of the sentence in which the equation appears. Thus an equation may be followed by a comma, semicolon, period, or no punctuation, depending upon its context. No space should be left between the last symbol of the equation and the punctuation mark except after a bar fraction, where one space should be left. Examples:

$$a = kr/(2k-rv^2).$$

$$a = \frac{kr}{2k-rv^2}.$$

(b) Punctuation Within a Line

If a single line of mathematics contains two or more short equations, or an equation and a remark expressed in symbols, the equations are generally separated by commas and/or words. There should be three or more spaces between the equations, so as to make a clear visual separation.

$$x^2 + y^2 + z^2 = 1, \quad ax + by + cz = 0.$$

$$\int_0^{\pi} \cos mx \cos nx \, dx = 0 \quad \text{if } m \neq n.$$

If there is not enough room for such a separation, the equations are displayed on two different lines.

Three dots (periods) are often used in mathematical work to stand for omitted symbols. Depending on context, the dots may be preceded or followed by other marks of punctuation, such as commas. If they are followed by a period, leave a space before the period to differentiate it from the dots; otherwise leave no space on either side.

$$a_{ij}x_j = b_i \quad (i = 1,2,\ldots,m;\ j = 1,2,\ldots,n),$$

$$J_0(x_i) = 0 \quad \text{for} \quad i = 1,2,\ldots .$$

"Broken definitions" are sometimes encountered in mathematical typing, usually in some recognizable variant of the following forms:

$$f(x) = \begin{cases} 0 & (x \leq \pi/2), \\ \cos x & (\pi/2 \leq x \leq \pi), \\ -1 & (x \geq \pi), \end{cases}$$

or

$$\begin{aligned} f(x) &= 0 && \text{if} \quad x \leq \pi/2, \\ &= \cos x && \text{if} \quad \pi/2 \leq x \leq \pi, \\ &= -1 && \text{if} \quad x \geq \pi. \end{aligned}$$

Observe in the first form the occurrence of the single brace, as an exception to the rule that braces always occur in pairs. Note also the alignment of the initial symbols of the expressions at the right, regardless of the positions of the inequality signs and of the varying lengths of the expressions on the left.

(c) Numbering of Equations

For reference purposes, an author may number some or all of his equations. For example,

$$x \sin x = 1, \qquad (3)$$

or

(3) $$x \sin x = 1.$$

The equation numbers should be written in a vertical column at the right or left margin, whichever is indicated by the author.

If an equation occupies more than one line and the equation number is written at the left, it should be on the first line of the equation; if at the right, on the last line. This includes equations which run on with successive = signs, as in Eq. (4) on page 50. If there is not room on the same line for the equation number to be clearly separated from its equation, the number should be written two spaces above the equation if on the left, or two spaces below if on the right.

Occasionally an author wishes to denote two or more equations by a single number. He may do this by dropping the equation number (assuming that it is on the right) below the whole group; or, if the equations are short, he may place the number midway in the margin opposite the equations.

$$\begin{aligned} x + y &= a, \\ x - y &= b. \end{aligned} \qquad (1)$$

6. Dividing Equations Between Lines

If an equation is too long to fit on one line, the author is the person who should decide where and how it is to be broken. Some authors do not do this, however, and so typists also need to know the principles according to which equations may be divided.

(a) Division at Equality Sign

The most satisfactory place to break a long equation is at an equality or inequality sign.

$$\int_{-\pi}^{\pi} \{\tfrac{1}{2}m + (m-1)\cos(x-t) + \ldots + \cos[(m-1)(x-t)]\}f(t)dt$$

$$= \frac{1}{2}\int_{-\pi}^{\pi} \frac{\sin^2\frac{1}{2}m(x-t)}{\sin^2\frac{1}{2}(x-t)}\, f(t)dt. \qquad (2)$$

In this example, the first line is longer than the second line, so the first line is centered and the second line is backspaced from the end of the first line.

If the second line is longer than the first line, then the second line is centered and the initial symbols of the two lines (not counting the = sign) are aligned vertically. The first line, no matter how short, is not to be indented farther than the second line.

$$z^4 + 2az^3 + bz^2 + 2acz + c^2$$

$$= \{z^2 + [a + (a^2+2c-b)^{\frac{1}{2}}]z + c\}\{z^2 + [a - (a^2+2c-b)^{\frac{1}{2}}]z + c\}. \tag{3}$$

If a sequence of equations runs on with successive = signs, many authors arrange the = signs in a vertical column, as follows:

$$\sum_{m,n=0}^{\infty}\left(\int_0^1 t^n(1-t)^m dt\right)\frac{x^m y^n}{m!n!} = \int_0^1 e^{xt}e^{y(1-t)}dt$$

$$= (e^x-e^y)/(x-y)$$

$$= \sum_{k=1}^{\infty}\frac{1}{k!}(x^k-y^k)/(x-y)$$

$$= \sum_{k=1}^{\infty}\frac{1}{k!}\{x^{k-1}+x^{k-2}y+\ldots+y^{k-1}\}. \tag{4}$$

However, if the = signs in the manuscript are very clearly not aligned, the typist should attempt to follow the arrangement of the manuscript.

(b) Division at Operation Signs (+, -, ×, ÷)

An equation may also be broken at a + or - sign.

$$\sigma^2 = \int_{-\infty}^{\infty} \{|Y_m|^2 N + |Y_m - e^{\alpha i\omega}|^2 S\} d\omega$$

$$+ \int_{-\infty}^{\infty} \{k^2(N+S)|\Delta_y|^2 + 2k[Y_m(N+S) - e^{\alpha i\omega} S]\tilde{\Delta}_y\} d\omega. \qquad (5)$$

Do not place a + or - sign directly beneath an = sign. If the first line is shorter than subsequent lines, indent the + or - sign two spaces beyond the = sign. If the first line is longer, backspace the remainder of the equation from the longest line.

If necessary (this is less desirable than the preceding) a product may be split.

$$p_n(x) = \frac{2^{-n/2}}{\Gamma(n/2)} - \frac{x}{(n-2)!2} + \frac{x^2 2^{-1-n/2}}{(n-2)!\Gamma[(n-2)/2]}$$

$$\times[- \log(2^{-n}x^2) + (- 3C - 2 \log 2 + 3)]. \qquad (6)$$

Here it is always necessary to supply a multiplication sign (× or ·) at the beginning of the second line. The rules for indenting or backspacing are the same as above.

Similarly, in emergencies a quotient may be split. For example,

$$-(\beta_0 a)^2 x^{-1} = ip\beta_0 a[(\varepsilon'-i\varepsilon''-1+\nu^2)\tan^2\psi + (\varepsilon'-i\varepsilon'')]/(\varepsilon'-i\varepsilon''-1+\nu^2)^{\frac{1}{2}}$$

is properly done in elite type, but if it had to be split it would go into

$$-(\beta_0 a)^2 x^{-1} = ip\beta_0 a[(\varepsilon'-i\varepsilon''-1+\nu^2)\tan^2\psi + (\varepsilon'-i\varepsilon'')] \div (\varepsilon'-i\varepsilon''-1+\nu^2)^{\frac{1}{2}},$$

using the sign ÷ rather than / to indicate division. The last equation would be more attractive if the author had used a horizontal bar in the first place, i.e.,

$$-(\beta_0 a)^2 x^{-1} = \frac{ip\beta_0 a[(\varepsilon'-i\varepsilon''-1+\nu^2)\tan^2\psi + (\varepsilon'-i\varepsilon'')]}{(\varepsilon'-i\varepsilon''-1+\nu^2)^{\frac{1}{2}}} ;$$

but the typist must not make such a rearrangement on her own.

If a long bar fraction is too long for a single line, enclose the numerator and denominator in sufficiently large brackets and connect them by a division sign.

$$\frac{\left[a\left(x^2 - x_1^2\right)\left(x^2 - x_2^2\right)\left(x^2 - x_3^2\right)\ldots\left(x^2 - x_k^2\right) + b\left(y^2 - y_1^2\right)\left(y^2 - y_2^2\right)\left(y^2 - y_3^2\right)\ldots\left(y^2 - y_\ell^2\right)\right]}{\div\left[c\left(z^2 - z_1^2\right)\left(z^2 - z_2^2\right)\left(z^2 - z_3^2\right)\ldots\left(z^2 - z_m^2\right)\right]}$$

The numerator and denominator may be divided, preferably at + or - signs, as described above. In such cases one may think of the numerator and denominator as separately centered above and below an imaginary horizontal bar. However, the use of a real horizontal bar, instead of a ÷ sign, to separate numerator and denominator in this example would not generally be considered good form, and should be done only if the author asks for it.

When an equation is divided, the connecting symbol (=, +, -, ×, or ÷) must be placed at the beginning of the second line. Some authors repeat the symbol at the end of the first line; or, if the break is at a - sign, they may supply a + sign at the end of the first line. In any particular case, the style of the individual manuscript should be followed.

(c) Division Within Parentheses, Brackets, or Braces

When possible, avoid splitting an expression within a pair of parentheses or other enclosing symbols. The following example is correctly divided.

$$c_6 = b_6 + 4\left(2b_1^2b_5 + 2b_2b_4\right) + 12\left(3b_1^2b_4 + 6b_1b_2b_3 + b_2^3\right)$$

$$+ 60\left(2b_1^3b_3 + 3b_1^2b_2^2\right) + 330b_1^4b_2 + 132b_1^6.$$

If it is necessary to divide a long expression within enclosing symbols, the break should preferably be made at a + or - sign. The second line must start to the

right of the left-hand enclosing symbol. In the following example, note how this rule is applied both to the outer braces and to the inner brackets.

$$\ddot{x}^{(2)} = -\frac{k}{r^{(0)3}}\left\{x^{(2)} - \frac{3x^{(1)}}{r^{(0)2}}\Xi + \frac{15x^{(0)}}{r^{(0)14}}\Xi^2\right.$$

$$-\frac{3x^{(0)}}{2r^{(0)2}}\left[x^{(1)2} + y^{(1)2} + z^{(1)2} + 2x^{(0)}x^{(2)}\right.$$

$$\left.\left. + 2y^{(0)}y^{(2)} + 2z^{(0)}z^{(2)}\right]\right\} + F_x^{(1)}.$$

(d) Undesirable Breaks

An equation should not be divided at the end of a page without consulting the author about it. If it is absolutely necessary to type a very long equation on two pages, the connecting sign (=, +, -, etc.) should be given both at the end of the first page and at the beginning of the second. One should also type the abbreviation "(cont.)" in the lower right corner of the first page, to indicate a continuation.

An expression which is short enough to be imbedded in the text, such as sin(x+y) or $a \geq b$, should never be divided between two lines. If it will not go entirely on the first line, it should all be put on the second. An author who objects to the resultant shortening of the first line should have displayed the expression on a separate

III. MATHEMATICAL SYMBOLS AND SPECIAL CHARACTERS

1. Enclosures, Assertions, and Operations

Symbol	Name
[	brackets
]	
$\langle$	angular brackets
$\rangle$	
{	braces
}	
⌜	special brackets
⌞	
⌝	special brackets
⌟	
$\approx$, $\sim$	approximately equal
$\equiv$	identically equal
$\neq$	not equal
$>$	greater than
$<$	less than
$\propto$	proportional (large)
$\propto$	proportional (small)
$\supset$	contains
$\subset$	contained in
ϵ	belongs to, is an element of
$\rightarrow$	forward arrow
$\leftarrow$	reverse arrow
$\times$	times
$\div$	divided by
$\cup$	union, cup
$\cap$	intersection, cap

2. Other Symbols

Symbol	Name
`	accent, grave
´	accent, acute
^	caret
¸	cedilla
ˆ	circumflex
~	cycle (also "approximately equals")
†	dagger
‡	dagger (double)
∇	del (nabla)
¨	diaeresis
≜	equilateral
Ħ	H-bar
ħ	h-bar
Ɨ	I-bar
∞	infinity
⌠	integration sign (large)
⌡	
∫	integration sign (small)
1	one
‖	parallel
∂	partial differential
⊥	perpendicular
′	prime
″	prime (double)
√	radical
/	solidus (slash)
§	section
~	tilde
|	vertical bar (centered)
~	wavy line

3. Double-Size Keys

Symbol	Name
$\prod$	capital pi (large)
$\sum$	capital sigma (large)
$($	large parentheses
$)$	
$[$	large brackets
$]$	
$\{$	large braces
$\}$	

4. Selectric Symbols

Mathematical

±	plus or minus
×	times
•	times
≡	identically equal
>	greater than
<	less than
∝	proportional
÷	divided by
∞	infinity
∇	del (nabla)
√	radical
()	parentheses; left, right
{}	braces; left, right
∫	integral
≃	approximately equals
ℓ	symbol "ell"
∿	cycle (also "approximately equals")
∂	partial differential
∑	capital sigma
⌠ ⌡	integration sign (large)

Chemical

→	give, pass over to, or lead to
←	forms and is formed
↓	indicates precipitation of the substance
↑	indicates that the substance passes off as a gas

Language Accents

´	acute
`	grave
¨	dieresis
~	tilde
^	circumflex

Miscellaneous

¶	paragraph
§	section
†	dagger
=	double underscore

Greek Alphabet

Upper Case	Lower Case		Upper Case	Lower Case		Upper Case	Lower Case	
	α	alpha		ι	iota		ρ	rho
	β	beta		κ	kappa	Σ	σ	sigma
Γ	γ	gamma	Λ	λ	lambda		τ	tau
Δ	δ	delta		μ	mu	Υ	υ	upsilon
	ε	epsilon		ν	nu	Φ	φ	phi
	ζ	zeta		ξ	xi		χ	chi
	η	eta		ο	omicron	Ψ	ψ	psi
Θ	θ	theta	Π	π	pi	Ω	ω	omega

Selectric Script

A	a
B	b
C	c
D	d
E	e
F	f
G	g
H	h
I	i
J	j
K	k
L	l
M	m
N	n
O	o
P	p
Q	q
R	r
S	s
T	t
U	u
V	v
W	w
X	x
Y	y
Z	z

1
2
3
4
5
6
7
8
9
0

5. <u>Selectric Transfers</u> (Consult Transcription for additional styles)

α β γ Δ δ θ λ μ

π ρ Σ ϕ ψ Ω

$\leq$ $\geq$ $\gg$ $\ll$ $\pm$ $\mp$ $\simeq$ $\approx$

$\sum$ Σ Σ $\sum$ $\prod$

$\int$ $\int$ $\int$ $\int$ $\int$ $\sqrt{}$ $\sqrt{}$ $\sqrt{}$

$( )$ $( )$ $( )$ $( )$

$[ ]$ $[ ]$ $[ ]$ $[ ]$

$\{ \}$ $\{ \}$ $\{ \}$